Транспорт в неравновесном окружении.

Mesoscopic transport in nonequilibrium environment

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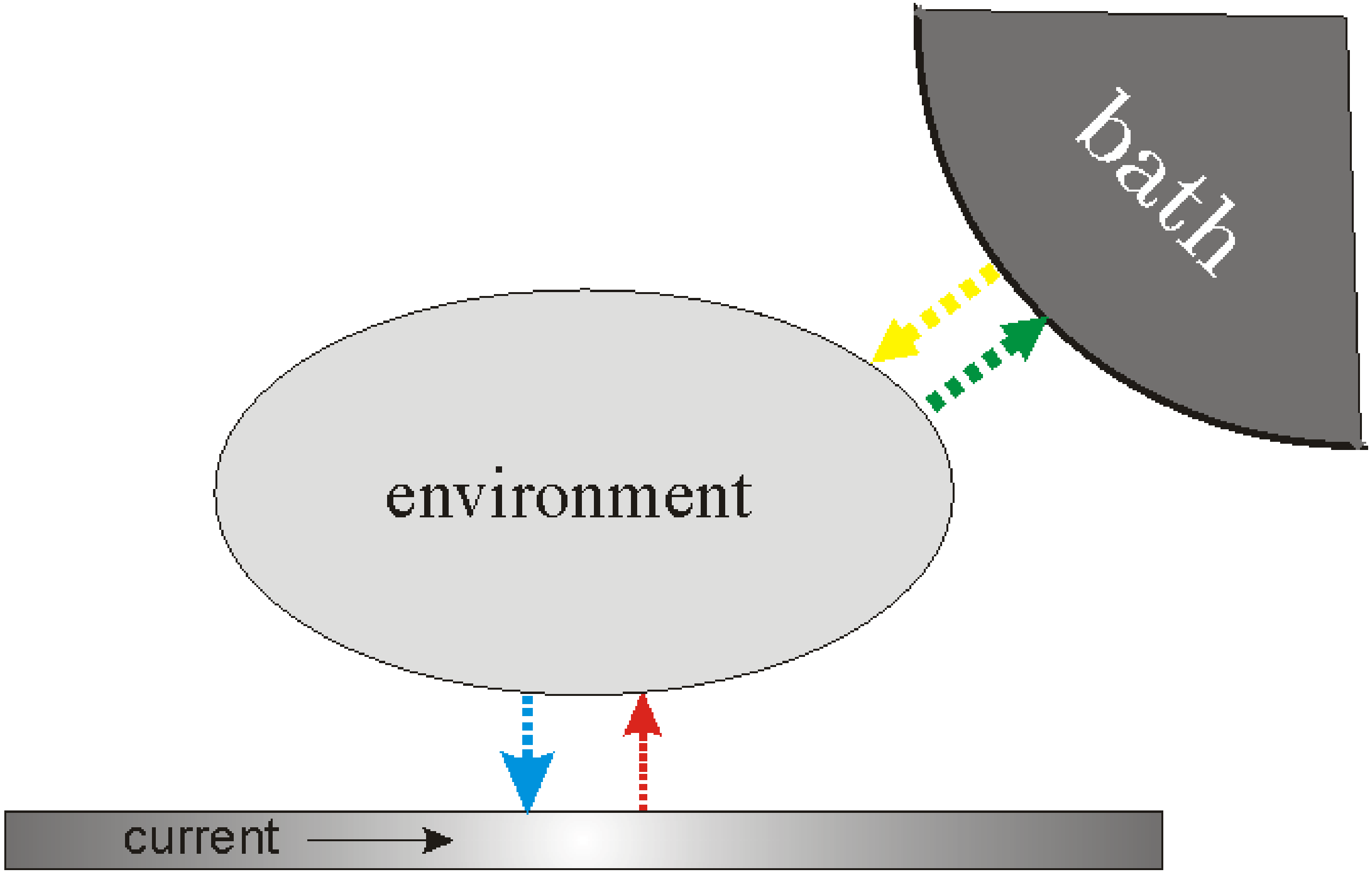
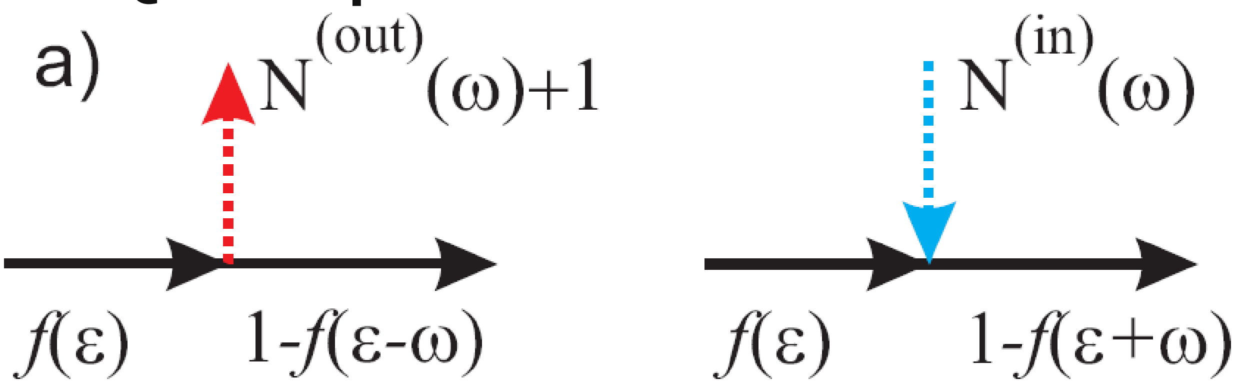
Plan

- Introduction. Old problems easily solvable using the concept of the environment.
- Chaos deblocade of tunneling current in arrays of Josephson junctions
- Tunnel junction in the nonequilibrium environment

- Environment and bath are different objects.
- Environment can become nonequilibrium.
- Environment can be tuned.



Quasiparticle relaxation



Scattering integral:

$$\frac{d\tilde{f}_e(\epsilon)}{dt} = -\int_\omega K^{(\rho)}(\omega,\epsilon)\{\tilde{f}_e(\epsilon)[1-\tilde{f}_e(\epsilon-\omega)][1+N_\omega] - [1-\tilde{f}_e(\epsilon)]\tilde{f}_e(\epsilon-\omega)N_\omega\}$$

Electron-electron scattering: $N_\omega = \frac{1}{\omega}\int [1-\tilde{f}_e(\epsilon_-)]\tilde{f}_e(\epsilon_+)d\epsilon$

Electron-phonon scattering: N_ω -- Phonon distribution function

Electron-environment scattering ????

$$T \ll 1/\tau$$

$$\frac{1}{\tau_T} = \frac{3\sqrt{3\pi}}{16}\zeta\left(\tfrac{3}{2}\right)(\sqrt{8}-1)\frac{\tau^{1/2}}{k_F l}\frac{T^{3/2}}{\mu\tau}$$

$$\mathrm{K}_3(\omega) = \frac{\omega}{8\sqrt{2}\pi^2 N_3(0) D^{3/2}}, \quad \omega\tau \ll 1$$

In the disordered limit $\lambda_{ph} \gg \ell_{el}$, the temperature dependence of the electron-phonon scattering rate is expected to follow either the T^2 or T^4 law, depending on the nature of the disorder (Sergeev and Mitin, 2000).

Mesoscopic transport in a nonequilibrium environment

[introduction]

- Far from equilibrium transport phenomena in mesoscopic conductors are usually governed by the energy exchange between the transport agents (e.g., electrons) and the environment of the bosonic excitations represented, by phonons, photons, many-body excitations (e.g., electron-hole pairs), the electromagnetic modes in the leads…

- ❑heating
- ❑the Coulomb anomaly
- ❑the Coulomb drag
- ❑the hooping (cotunneling)
- ❑the many body localization
- ❑the resonance exciting of the environment modes
- ❑Spin resonance (e.g., NMR)
- ❑…

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the resonance exciting of the environment modes

Coupled Superconducting Phase and Ferromagnetic Order Parameter Dynamics

I. Petković[1*], M. Aprili[1], S. E. Barnes[2,3], F. Beuneu[4], and S. Maekawa[5,6]

Via a direct coupling between the magnetic order parameter and the singlet Josephson supercurrent, we detect spin-wave resonances, and their dispersion, in ferromagnetic Josephson junctions in which the usual insulating or metallic barrier is replaced with a weak ferromagnet. The coupling arises within the Fraunhofer interferential description of the Josephson effect, because the magnetic layer acts as a time dependent phase plate. A spin-wave resonance at a frequency ω_s implies a dissipation that is reflected as a depression in the current-voltage curve of the Josephson junction when $\hbar\omega_s = 2eV$. We have thereby performed a resonance experiment on only 10^7 Ni atoms.

arXiv:0904.1780v1 [cond-mat.supr-con] 11 Apr 2009

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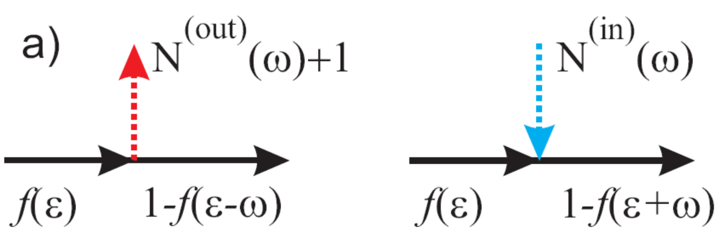
heating

A.F. Volkov and S.M. Kogan, UFN 96, 633 (1968)

a)



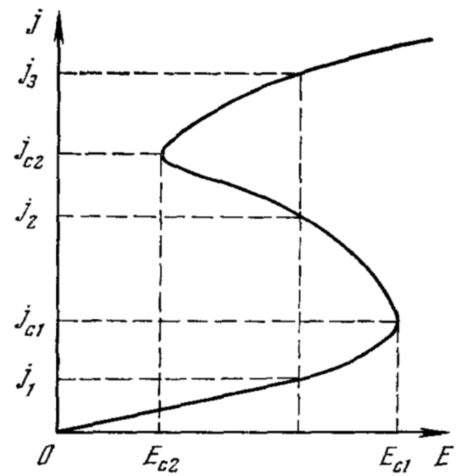


Рис. 1. S-образная вольт-амперная характеристика.

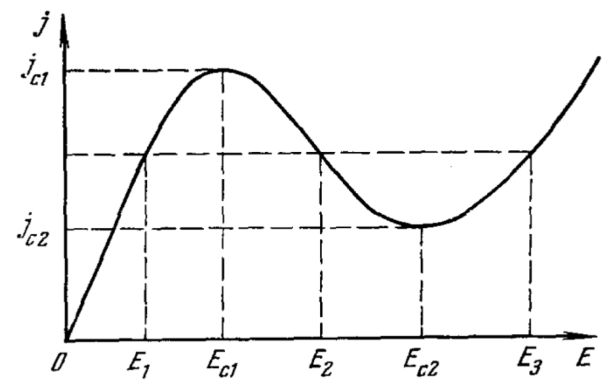


Рис. 2. N-образная вольт-амперная характеристика.

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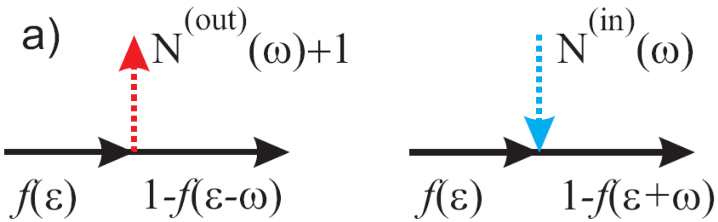
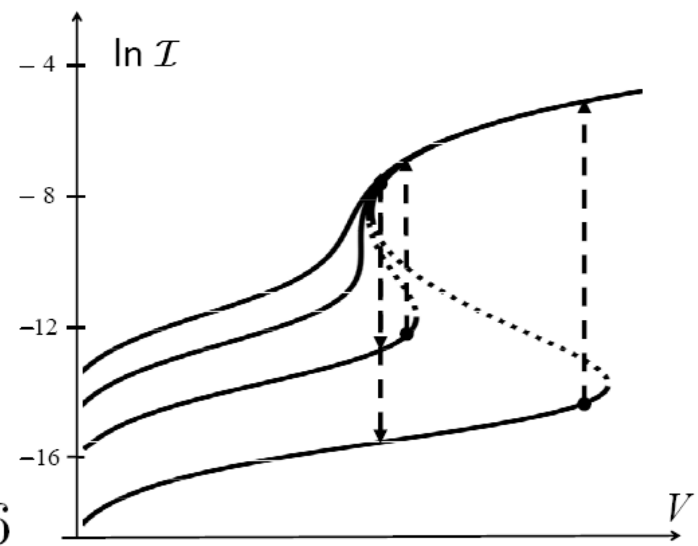
heating

A.F. Volkov and S.M. Kogan, UFN 96, 633 (1968);
rediscovered in B. Altshuler, et al, PRL 2009.



$$\frac{V^2}{R(T_{\rm el})} = \frac{\mathcal{E}(T_{\rm el})}{\tau_{\text{e-ph}}(T_{\rm el})} - \frac{\mathcal{E}(T_{\rm ph})}{\tau_{\text{e-ph}}(T_{\rm ph})},$$

$$\frac{1}{\tau_{\text{e-ph}}(T)} = \left(\frac{T}{\hbar}\right)\left(\frac{T}{\Theta}\right)^{\beta-3}, \quad \mathcal{E} = \pi^2\nu\mathcal{V}T^2/6$$



Mesoscopic transport in a nonequilibrium environment

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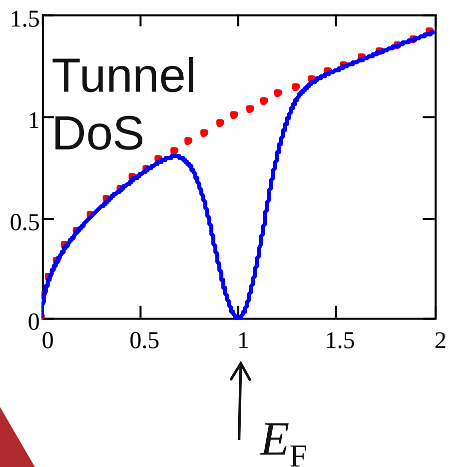
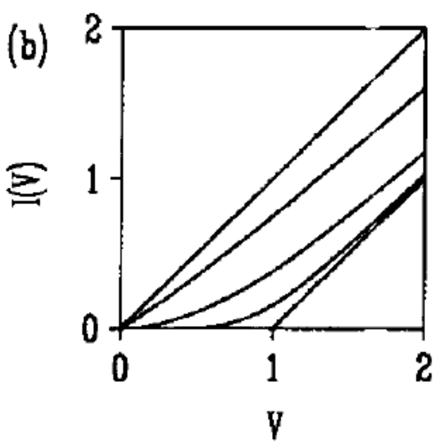
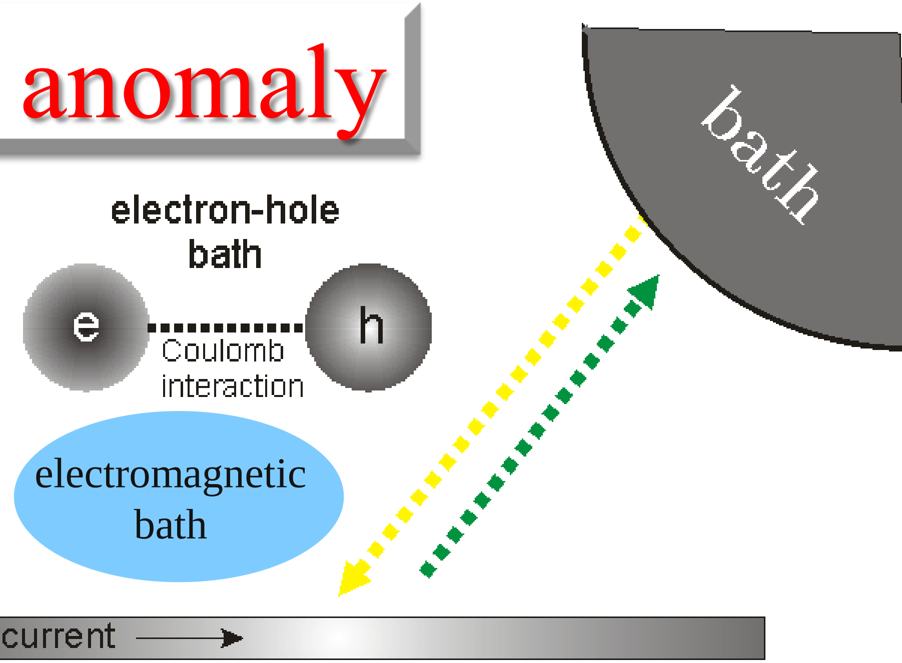
the Coulomb anomaly





Single Charge Tunneling, ed. by Grabert and Devoret 1991



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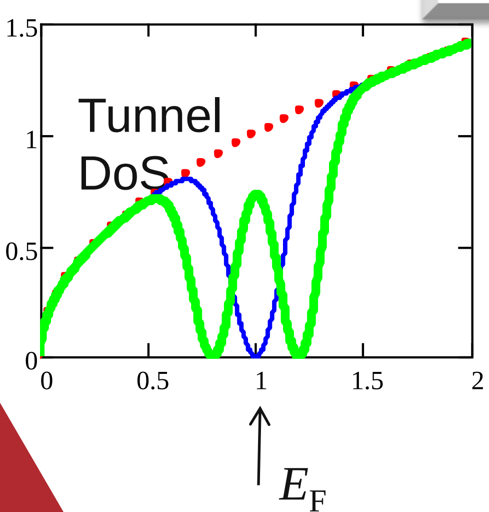

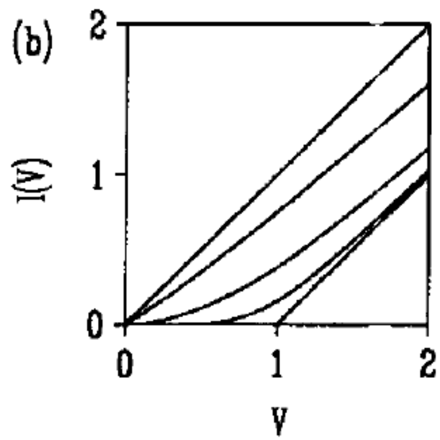

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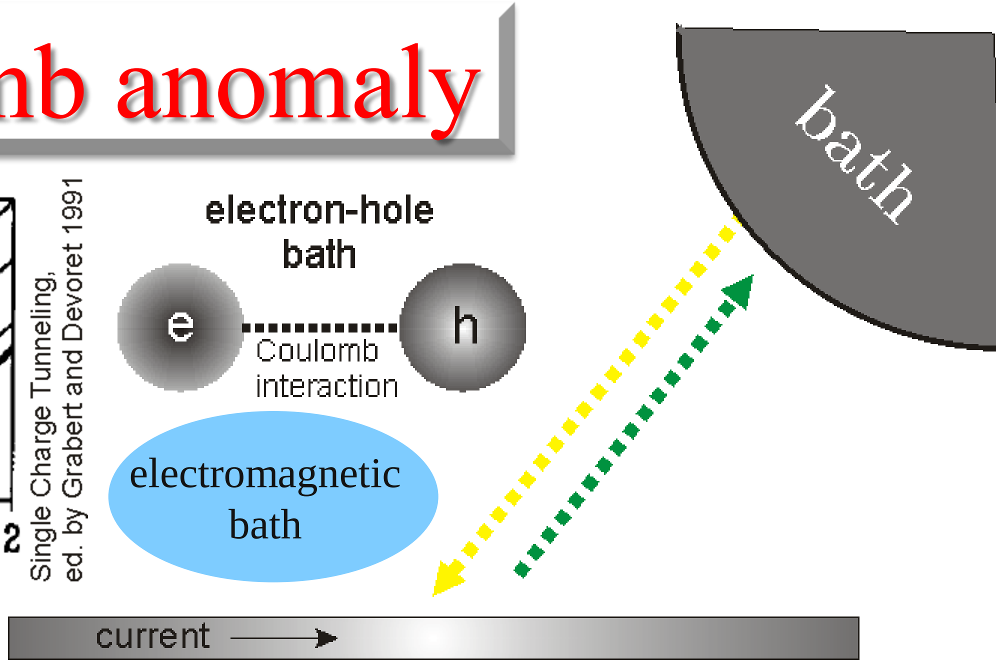

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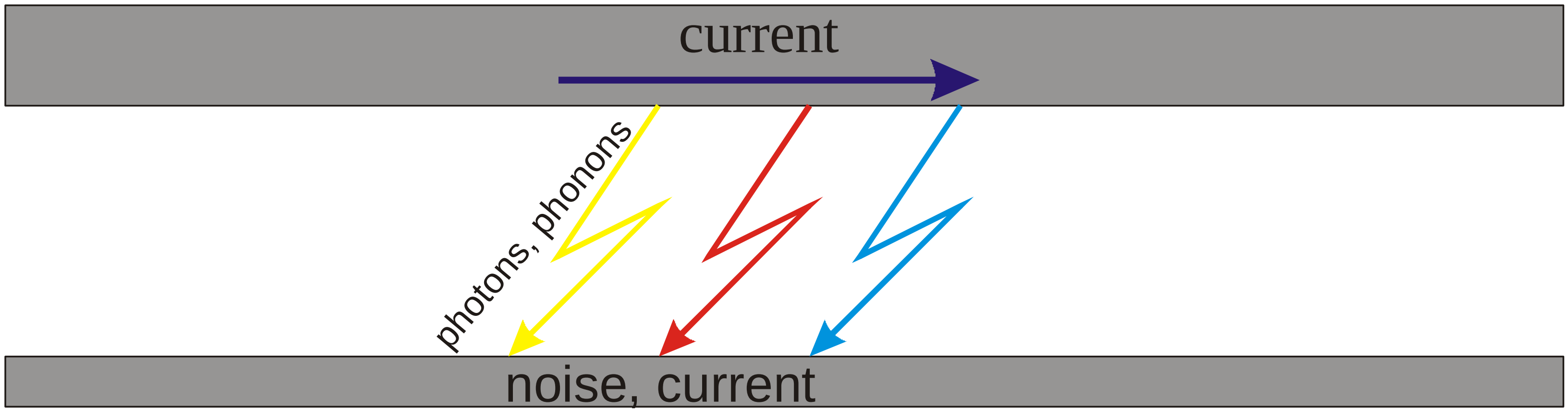
the Coulomb drag



Mesoscopic transport in a nonequilibrium environment

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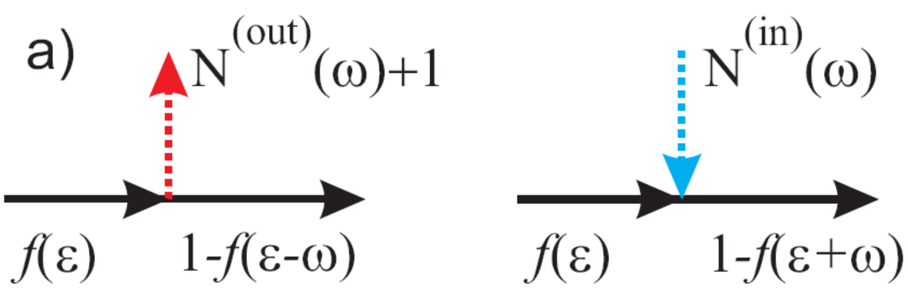
hooping transport

Nearest center hooping in

impurity-band insulators/semiconductors.

$$\rho = \rho_{b0} \exp \frac{\varepsilon_b}{T}$$



Variable range hooping in
impurity-band insulators/semiconductors.

$$\rho = \rho_0 \exp \left(\frac{T_M}{T} \right)^{1/4}$$

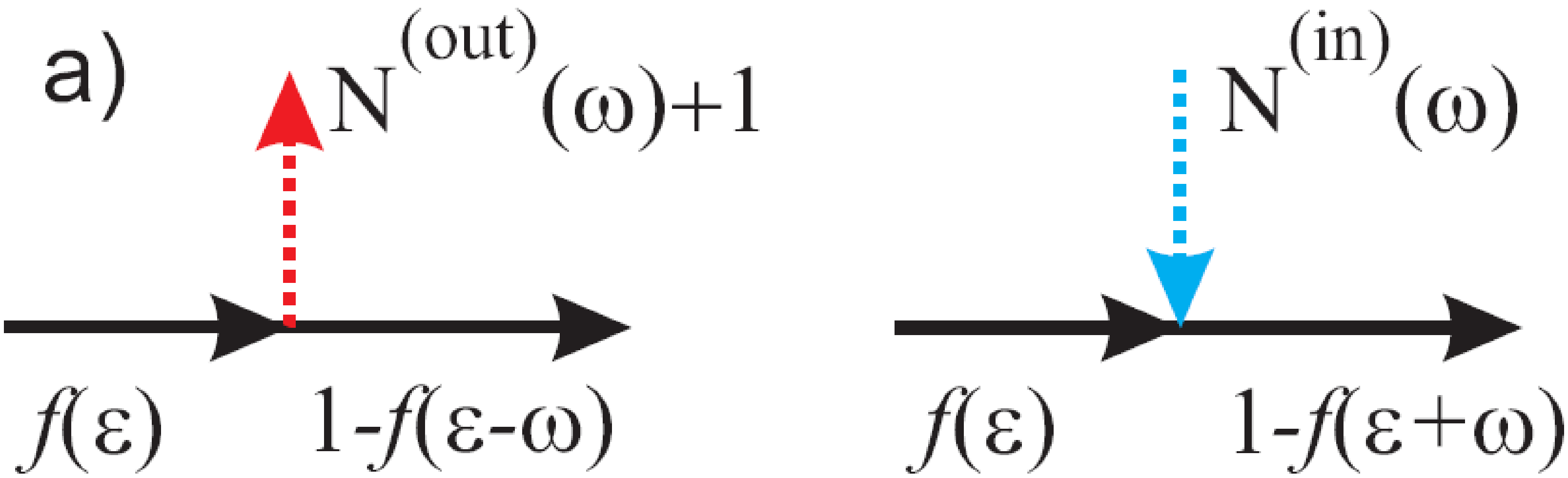
$$\rho = \rho_0 \exp \left(\frac{T_{ES}}{T} \right)^{1/2}$$

Mesoscopic transport in a nonequilibrium environment

<u>The concept of an environment gives an opportunity:</u>

✓ to unite different from the first glance physical phenomena,

✓ to develop the understanding of new physical problems.

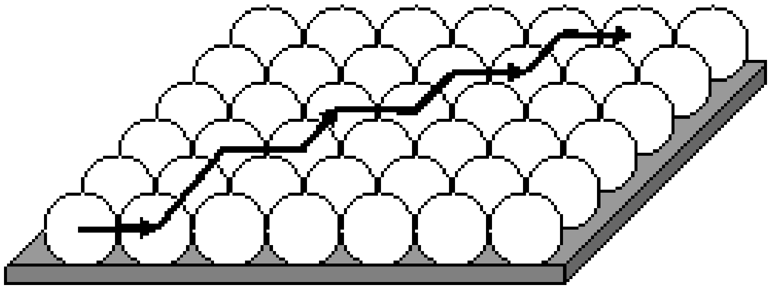


Example – “Many-body localization”.

Hopping conductivity in tunnel junction arrays

Efros-Shklovskii (or Mott)-type VRH should not be seen!!

arrays of granules: electrons have to tunnel through other granules



Hopping between the nearest neighbors, and no room for the hopping distance optimization

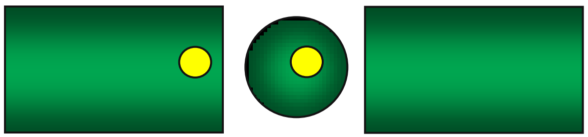
tunneling via virtual states of intermediate grains

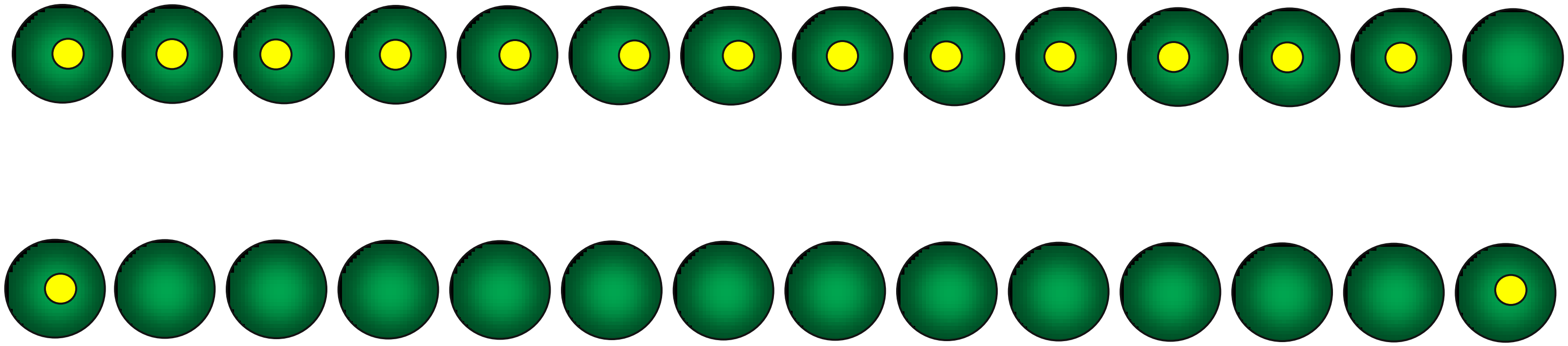
Elastic cotunneling mechanism $T < \sqrt{E_0^c \delta}$

Inelastic cotunneling mechanism $T > \sqrt{E_0^c \delta}$

Cotunneling

Coherent propagation





I. Beloborodov, A. Lopatin, V. Vinokur, PRB

Arrays: normal metal granules (N), Superconducting granules (S), Hybrid structures (SNS)…

And yet another puzzle of the VRH: the mechanism of relaxation…

$$\boxed{\sigma \propto \sigma_0 \exp\left[-\left(\frac{T_0}{T}\right)^{1/2}\right]}$$

$$\sigma_0 \approx \frac{e^2}{\hbar}$$

e-e interactions

Semiconductors:

S.I. Khondaker et al., Phys. Rev. B **59** 4580 (1999)

I. Shlimak et al., Solid State Commun. **112** 21 (1999)

A. Ghosh et al., Phys. Status Solidi B **230** (2002) 211.

Granular superconductors:

T.I. Baturina et al., Phys. Rev. Lett. **99,** 257003 (2007)

T.I. Baturina et al., Physica C **468,** 316 (2008)

T.I. Baturina et al., JETP Lett. **88,** 752(2008)

General results for array comprising of *N* junctions

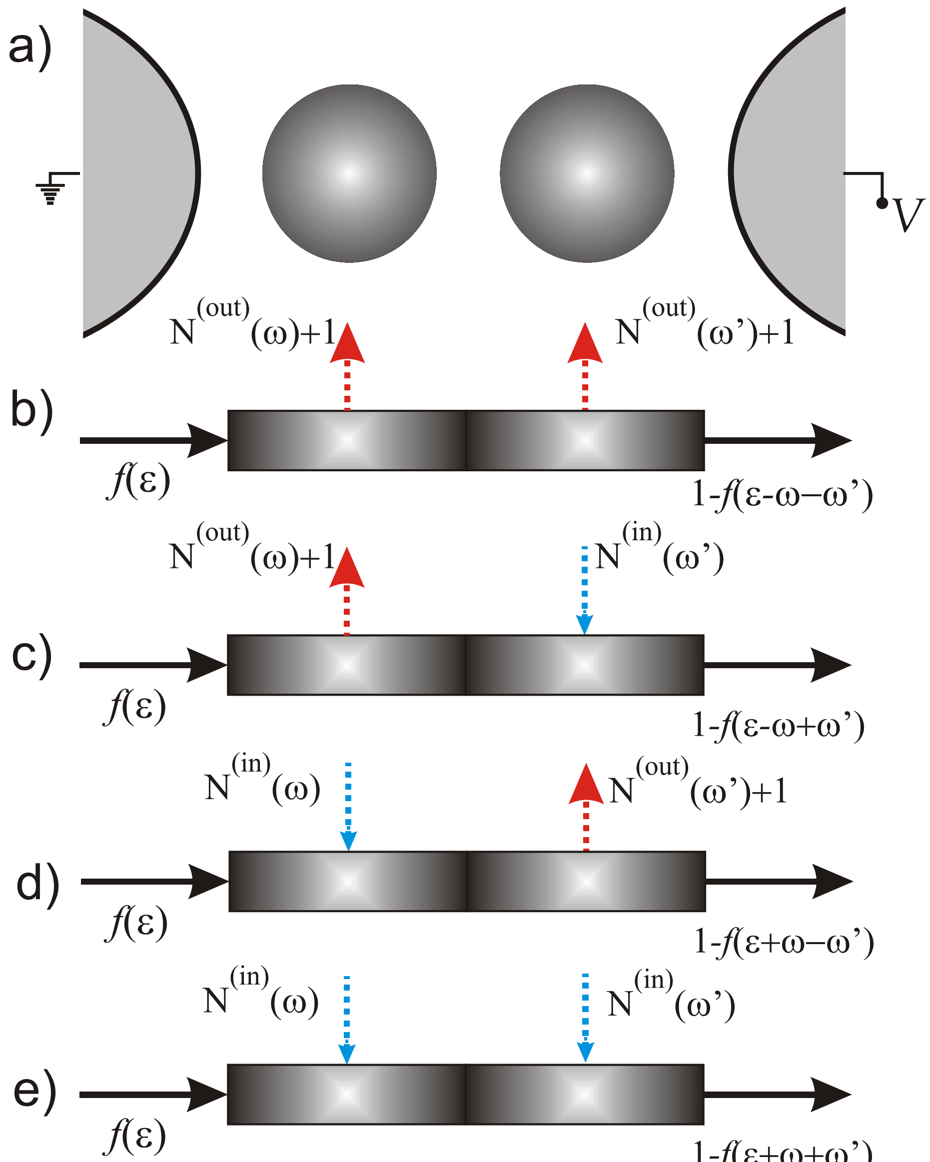
$$\overrightarrow{\Gamma} = \left(\prod_{i=1}^{N} \frac{R_K}{4\pi^2 R_i}\right) S^2 \int d\epsilon d\epsilon' f_1(\epsilon)[1 - f_2(\epsilon')]P(\epsilon - \epsilon')$$

$$P(E) = \int_{-\infty}^{\infty} dt \exp[iEt] \left\{ \int_0^{\infty} d\omega \frac{\rho(\omega)}{\omega} \times \prod_{j \le N-1} \left[N_{\omega,j}^{(\text{in})} e^{i\omega t} + (1 + N_{\omega,j}^{(\text{out})}) e^{-i\omega t} \right] \right\} .$$

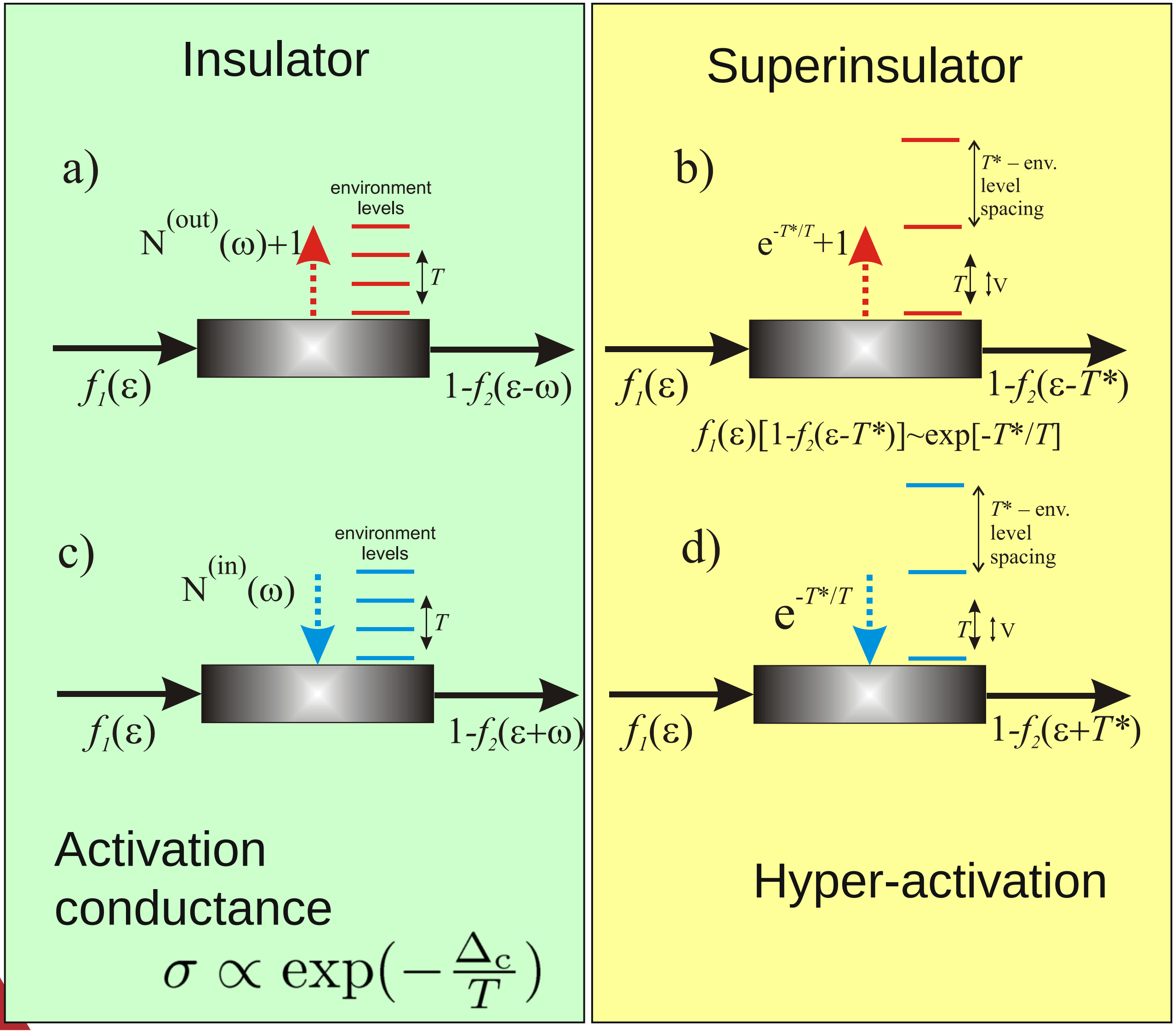
$$\boxed{I = e\left(\overrightarrow{\Gamma} - \overleftarrow{\Gamma}\right)}$$

Cotunneling

a) The single electron circuit with the two islands. Diagrams b)-e) correspond to the forward inelastic cotunneling rate through the structure. The red dotted lines show the electron-hole pairs that are excited during the cotunneling and the blue dotted lines show the annihilated e-h pairs. The vertices are shown by boxes. They are proportional to ρ/ω -- the probability of electron-hole pairs excitation during the charge transfer.



Corollaries: 2. Microscopic mechanism of Insulator-Superinsulator transition



In two dimensions the electron-hole plasma experiences the charge Berezinskii-Kosterliz-Thouless transition (BKT) where the gap of the order of the charging energy E_c appears in the environment excitations spectrum. Opening this gap completely impedes both Cooper pairs- and normal quasiparticle currents in the superconducting tunneling array at $T<E_c$. This is the microscopic mechanism of the insulator-to-superinsulator transition.

Corollaries: 1. Many-body localization

Finkel'stein action in Keldysh:

$$iS_F = -\frac{\sigma_N}{4}\,\mathrm{tr}\left[(\check{\partial}_r Q)^2 - \frac{4}{D}\check{\partial}_t Q\right] - \frac{iF_{\rm int}}{4\nu}\int_{tx}(\hat{\rho}_1)_{tx}(\hat{\rho}_2)_{tx} + 2i\nu\,\mathrm{tr}[\overrightarrow{\phi}^{\,T}\sigma_x\overrightarrow{\phi}]$$

$$F_{\rm int} = F_\rho(1+F_\rho)$$

(classical) (quantum)

density fluctuation

contact Coulomb interaction

(singlet channel)

$$\hat{\rho}_1 = -\frac{2\pi\nu}{1+F_\rho}\left\{\mathrm{tr}\left(\sigma_x Q_{t',t'}\right) + \frac{\phi_1}{2\pi}\right\}, \qquad \hat{\rho}_2 = -\frac{2\pi\nu}{1+F_\rho}\left\{\mathrm{tr}\left(Q_{t',t'}\right) + \frac{\phi_2}{2\pi}\right\}$$

THE FLUCTUATION PROPAGATOR

$$\mathcal{D}^{(R)} \sim \frac{1}{Dq^2 - i\omega[1+\Gamma_\rho]}$$

$$\Gamma_\rho = F_\rho/(1+F_\rho)$$

Corollaries: 1. many-body localization

$$\mathcal{D}^{(R)} \sim \frac{1}{Dq^2 - i\omega[1+\Gamma_\rho]}$$

$$Dq^2, \omega, \omega\Gamma_\rho \gg \delta \qquad \omega = 2\pi T n$$

$$\text{we get from } \delta \ll T\Gamma_\rho$$

"Many body localization"

$$\boxed{T \gg T^* = \delta/\Gamma_\rho}$$

$$\boxed{\Gamma_\rho = F_\rho/(1+F_\rho)}$$

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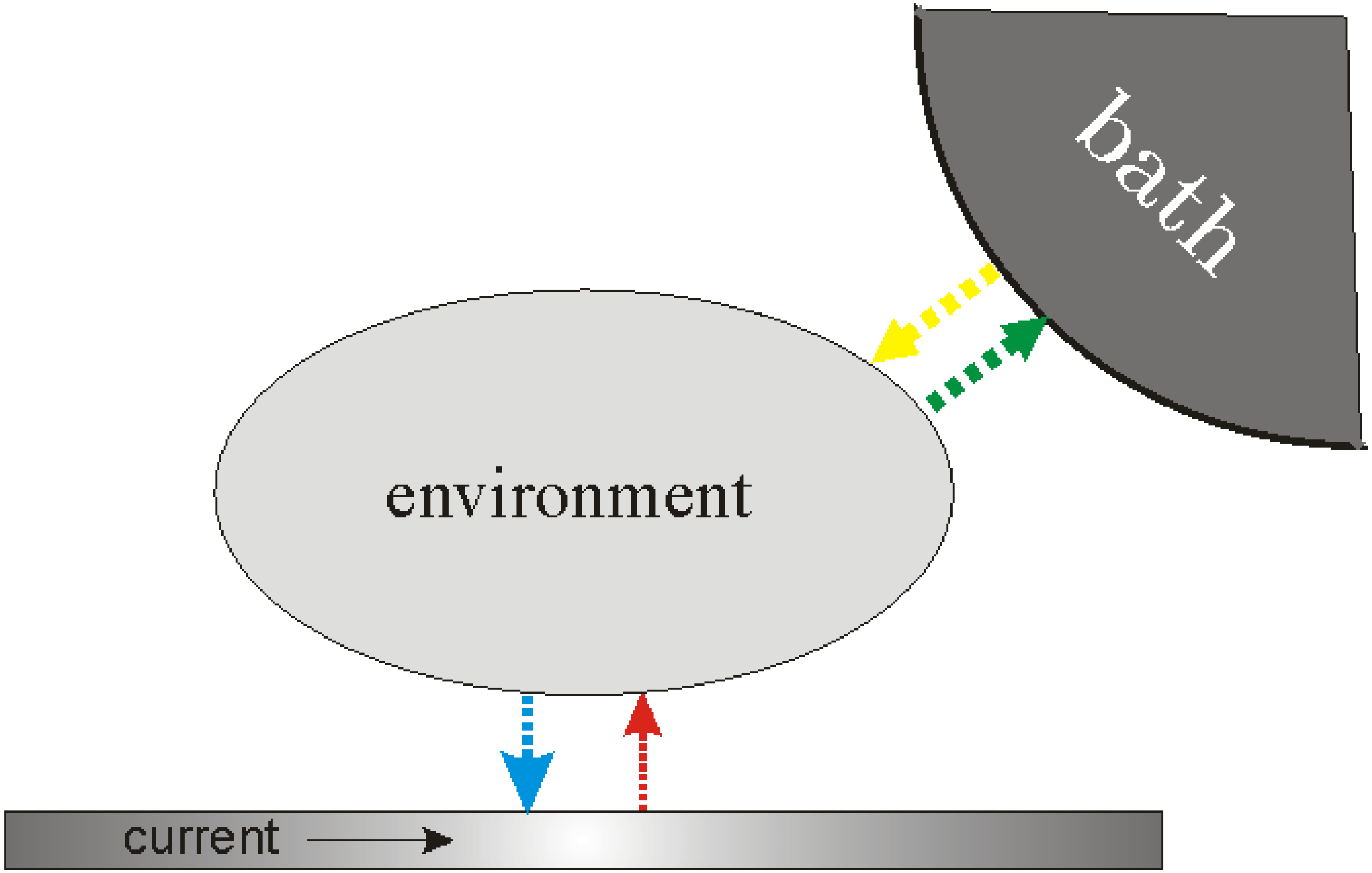
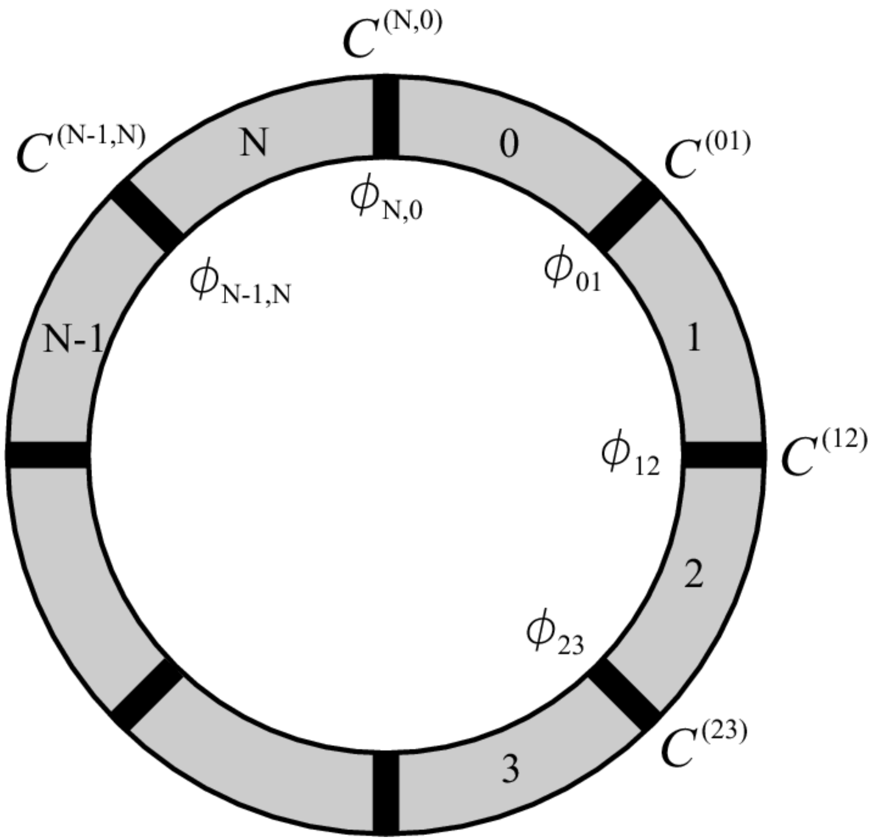
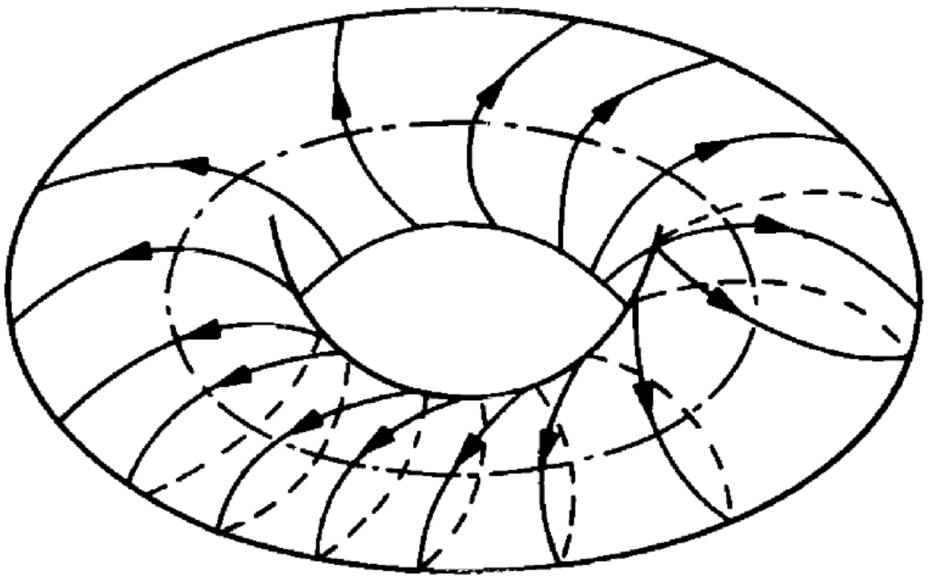


1D array of Josephson junctions.





Турбулентность Ландау-Хопфа

$$I_{\mathrm{qp}}(V) = \frac{1}{eR_T}\int_{-\infty}^{\infty} dE dE' N_S(E) N_S(E') P(E-E') \times$$
$$\{f_i(E)[1 - f_{i+1}(E')] - f_{i+1}(E)[1 - f_i(E')]\},$$

$$\boxed{I_s = -\frac{2E_J^2}{\hbar^2}\int_{-\infty}^{\infty} dt \sin[2eVt]\, \mathrm{Im}[\mathcal{K}(t)]}$$

$$\mathcal{K}(\omega) = \sum_{\{n\},\{m\}} P_{\{n\}} \left|\langle\{n\}|e^{i2\phi_{i,i+1}}|\{m\}\rangle\right|^2 \times$$
$$2\pi\delta(E_{\{n\}} - E_{\{m\}} + \omega)$$

$$P_{\{n\}} = Z^{-1} e^{-\sum_{i>0} \frac{E_c^{(i,i+1)} n_{i,i+1}^2}{2T}}$$

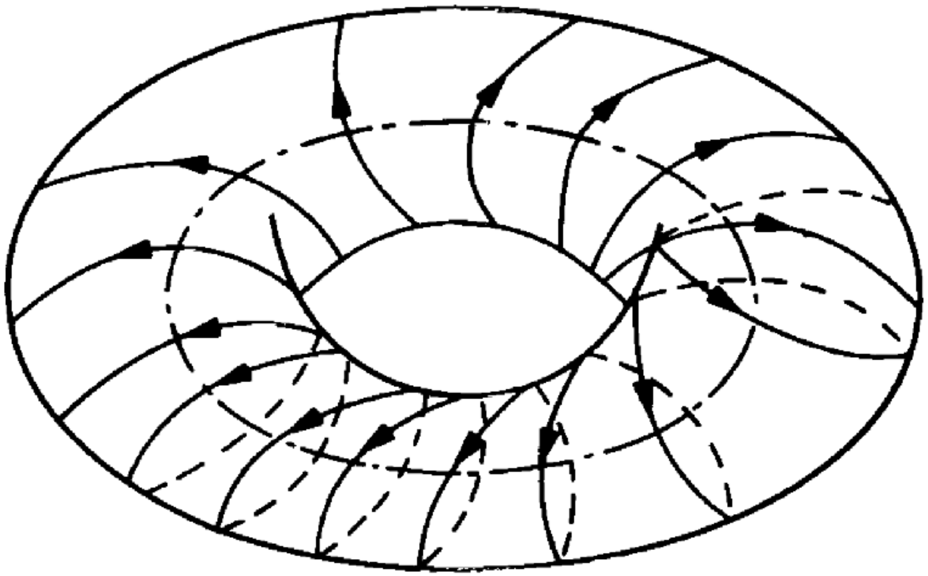
The quantum states of the annular JJAs have the wave function

$$|\{n\}\rangle = \prod_{i=0}^{N} \psi_{n_{i,i+1}}(\phi_{i,i+1}) \ ,$$

where $\sum_{i=0}^{N} \phi_{i,i+1} = 0$ and $\psi_n(\phi) = \exp\{in\phi\}/\sqrt{2\pi}$ is the wave function of the quantum rotator.

$$\mathcal{K}(t) = e^{i\Omega t} \sum A_{p_1 p_2 \ldots p_N} \exp\left\{ i \sum_{i=1}^{N} p_i \, \varphi_i(t) \right\}$$

$$A_{p_1 p_2 \ldots p_N} \; = \; P_{\{p\}} \; = Z^{-1} e^{-E_c^{(i,i+1)} p_i^2 / 2T}$$



$$I_s = D\left\{e^{-\frac{(V+N\bar{E}_c)^2}{2TN\bar{E}_c}} - e^{-\frac{(V-N\bar{E}_c)^2}{2TN\bar{E}_c}}\right\}\vartheta\left(z,\mathcal{T}\right)$$

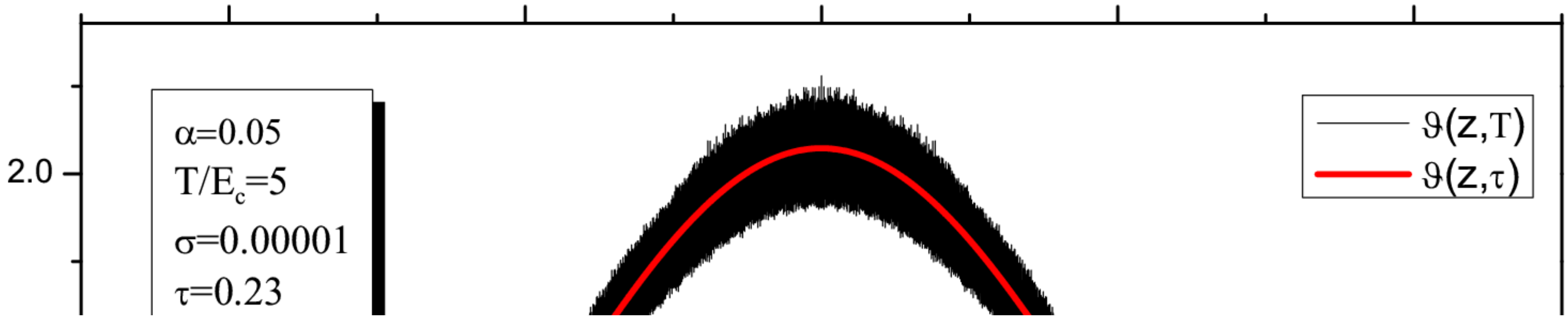
where $D = \frac{\pi E_J^2\sqrt{(2\pi T)^{N-1}}}{Z\sqrt{N\bar{E}_c^{N+1}}}$, and $z = V/\bar{E}_c$

$$\boxed{\vartheta(z,\mathcal{T}) = \sum_{\vec{m}} e^{i2\pi z\vec{m}\cdot\vec{a}-\pi\vec{m}^{\tau}\mathcal{T}\vec{m}}}$$

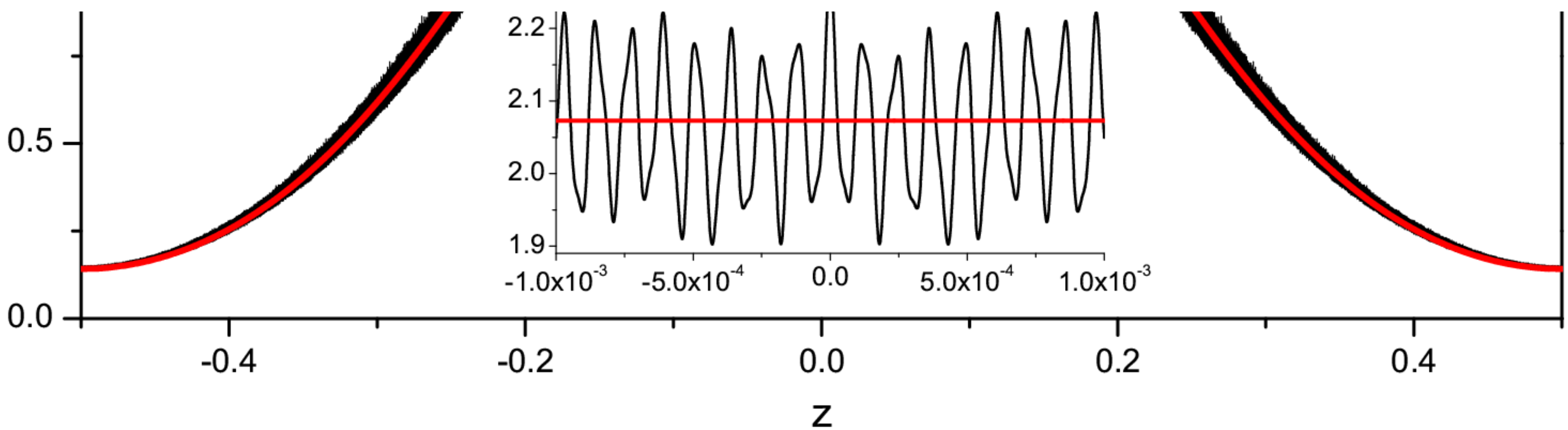
$$\mathcal{T}_{ij} = \frac{2\pi T}{\bar{E}_c}H_{ij} + \frac{\pi\sigma^2}{2(N\bar{E}_c)^2}, \quad H_{ij} = \frac{1}{N}\left(\frac{\sum e}{e_i}\delta_{ij} - 1\right)$$

Here $\sum e = \sum_k e_k$.

$$\vartheta(z,\tau) = \sum_{n=-\infty}^{\infty} e^{i2\pi n z - \pi n^2 \tau}, \quad \tau = 2\pi\left[\frac{\sigma^2}{4\bar{E}_c^2} + \frac{T\alpha^2 N}{\bar{E}_c}\right]$$



$$\alpha \bar{E}_c \lesssim \sigma e^S, \qquad S \approx Y N \ln N,$$



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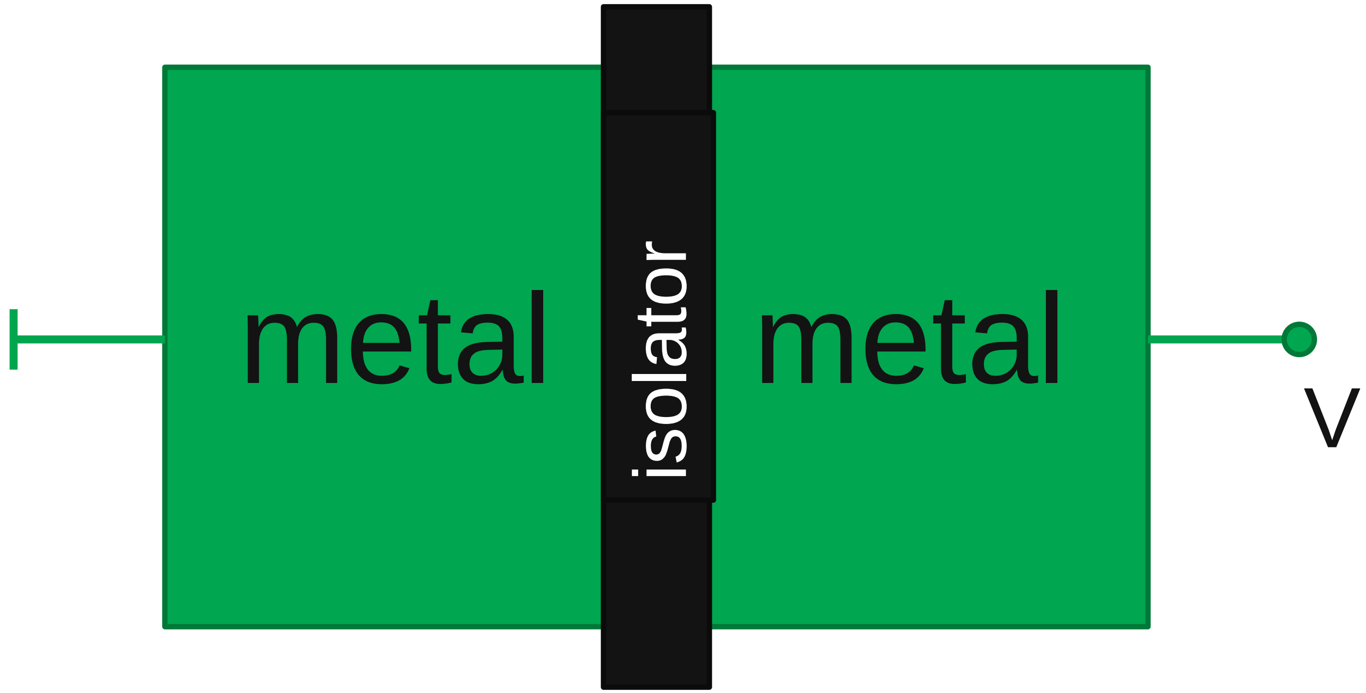
where $D = \frac{\pi E_J^2\sqrt{(2\pi T)^{N-1}}}{Z\sqrt{N\bar{E}_c^{N+1}}}$, and $z = V/\bar{E}_c$

$$\mathcal{K}(t) = e^{i\Omega t}\sum A_{p_1p_2\ldots p_N}\exp\left\{i\sum_{i=1}^{N} p_i\,\varphi_i(t)\right\}$$

$$A_{p_1p_2\ldots p_N} \;=\; P_{\{p\}} \;= Z^{-1}e^{-E_c^{(i,i+1)}p_i^2/2T}$$

Tunnel junction



$$\vec{\Gamma}(V) = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} \mathrm{d}E \mathrm{d}E' \, f(E)[1 - f(E' + eV)] P(E - E')$$

$$I(V) = V/R_T$$

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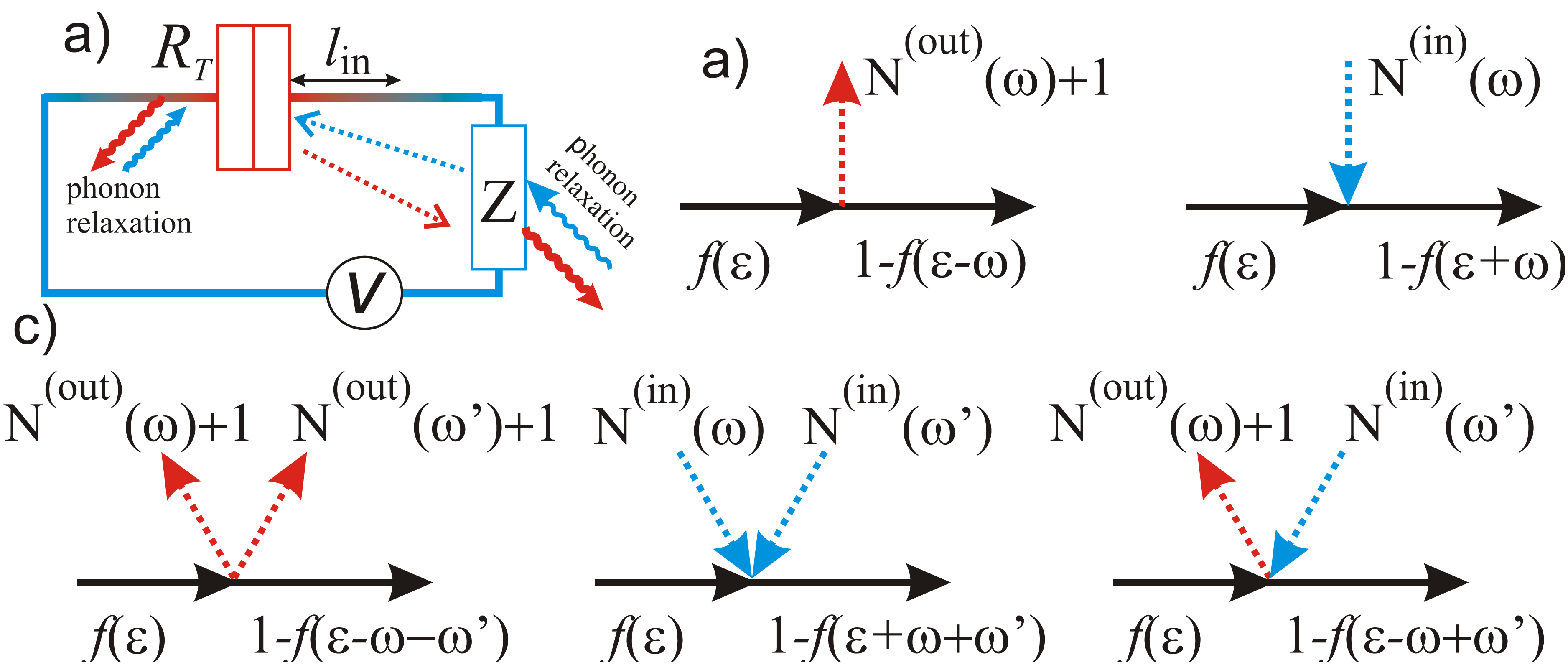


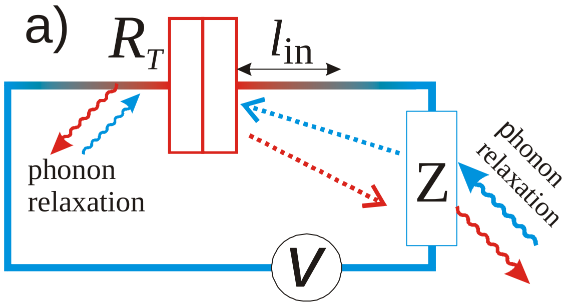
General results

$$I = e\left(\overrightarrow{\Gamma} - \overleftarrow{\Gamma}\right)$$

where $\overrightarrow{\Gamma}$ is the tunneling rate from left to the right

$$\overrightarrow{\Gamma} = \frac{1}{R_\mathrm{T}} \int_{\epsilon\epsilon'} f^{(1)}_{\epsilon}(1 - f^{(2)}_{\epsilon'}) P^{<}(\epsilon - \epsilon')$$

$$P^{<}(E) = \int_{-\infty}^{\infty} dt \exp[J(t) + iEt] \,,$$



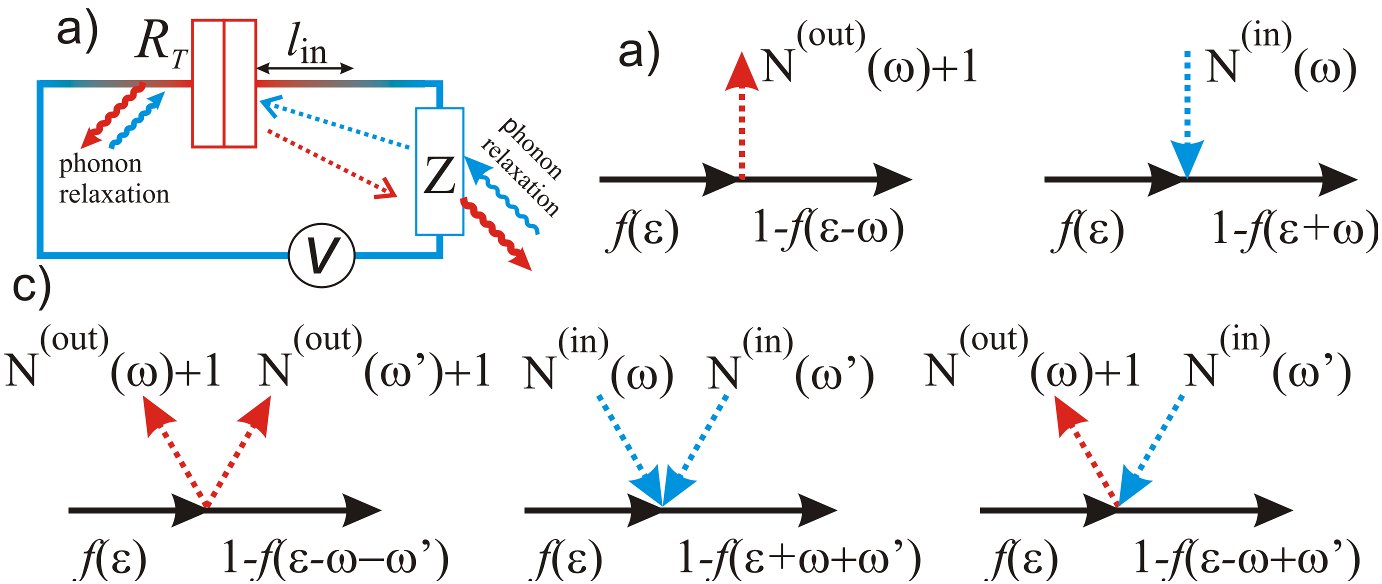
$$J(t) = 2\int_0^\infty \frac{d\omega}{\omega} \rho_\alpha(\omega) F_\alpha(\omega) \,,$$

$$F_\alpha(\omega) = \left[N_\omega^{(\alpha)} e^{i\omega t} + (1 + N_\omega^{(\alpha)}) e^{-i\omega t} - B_\omega^{(\alpha)}\right] ,$$

$N_\omega^{(\alpha)}$ and $1 + N_\omega^{(\alpha)}$ combinations of the environmental excitations distribution functions describe absorbed and emitted environmental excitations and the index

$$B_\omega^{(\alpha)} = [1 + N_\omega^{(\alpha)}] + N_\omega^{(\alpha)}$$ Equilibrium: $$B_\omega = \coth(\omega/2T)$$

Kinetic equations

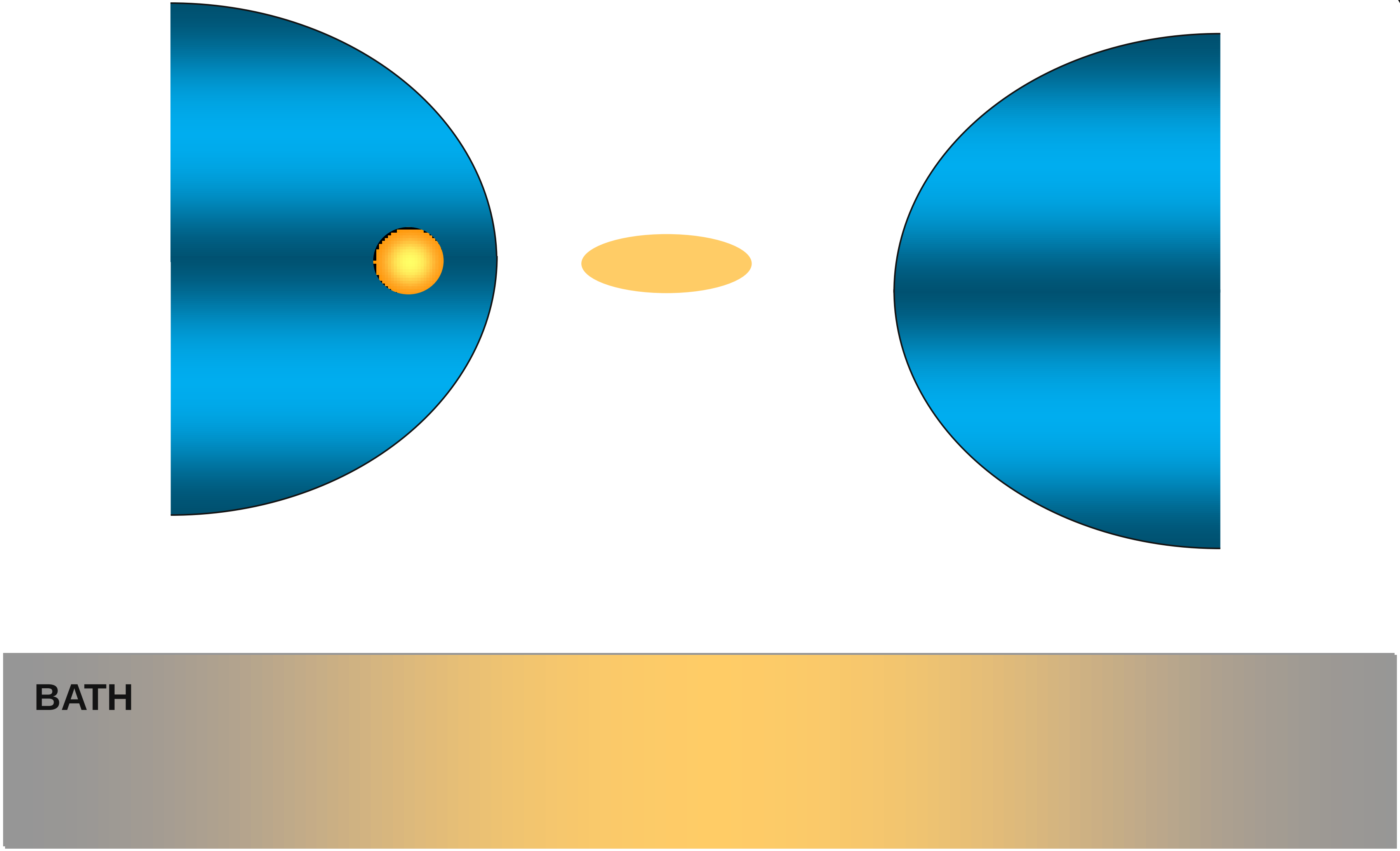


$$I^{(1)}_{\rm col} = -\int \nu_\omega d\omega \nu_2 d\epsilon_2 \left(\frac{\rho}{\omega\nu_\omega}\right) W_{12}\Big\{$$

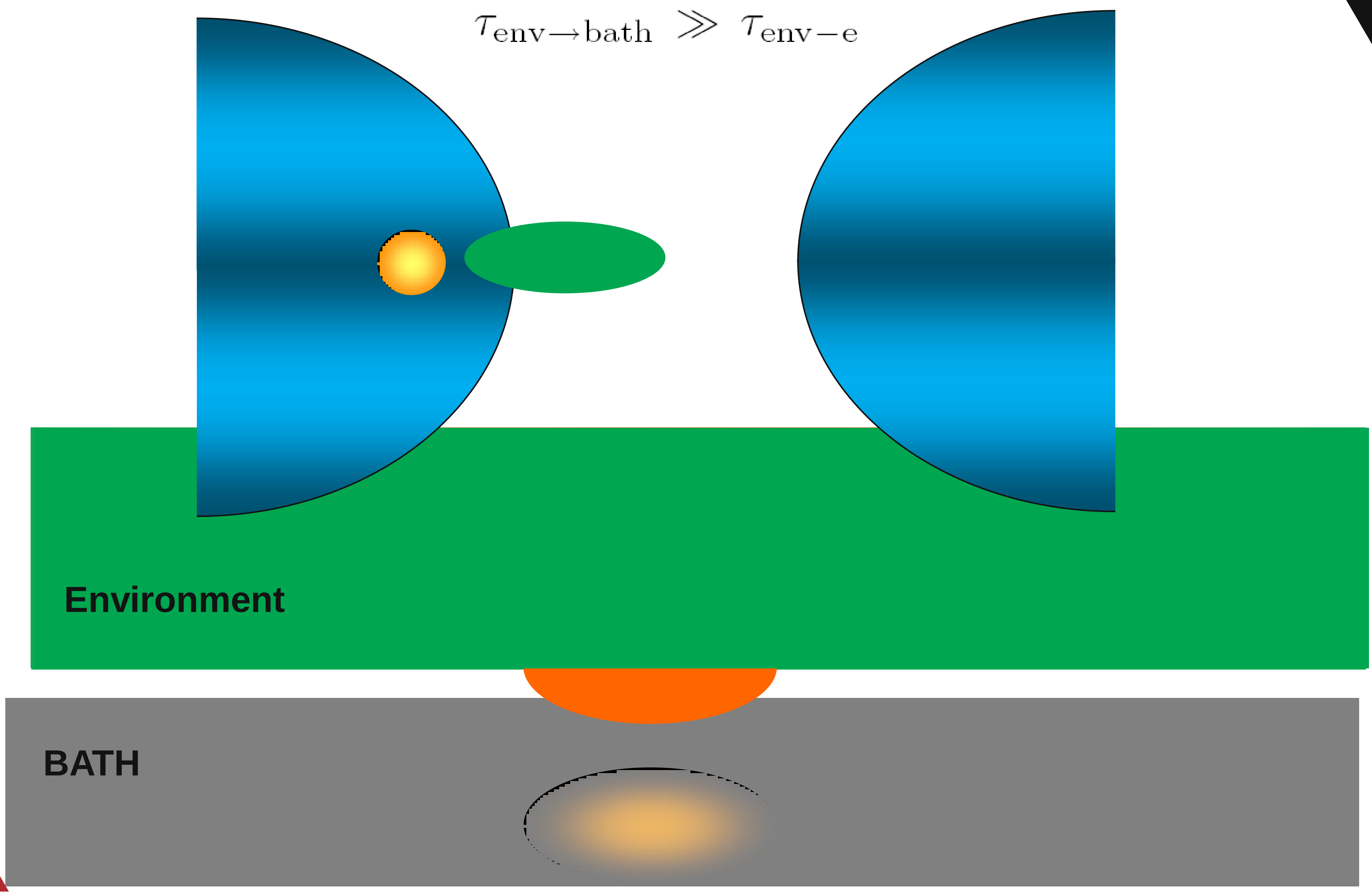
$$\delta(\epsilon_{12}-\omega)[f^{(1)}_{\epsilon_1}(N_\omega+1)(1-f^{(2)}_{\epsilon_2}) - (1-f^{(1)}_{\epsilon_1})N_\omega f^{(2)}_{\epsilon_2}]+$$

$$\delta(\epsilon_{12}+\omega)[f^{(1)}_{\epsilon_1}N_\omega(1-f^{(2)}_{\epsilon_2})-(1-f^{(1)}_{\epsilon_1})(N_\omega+1)f^{(2)}_{\epsilon_2}]\Big\},$$

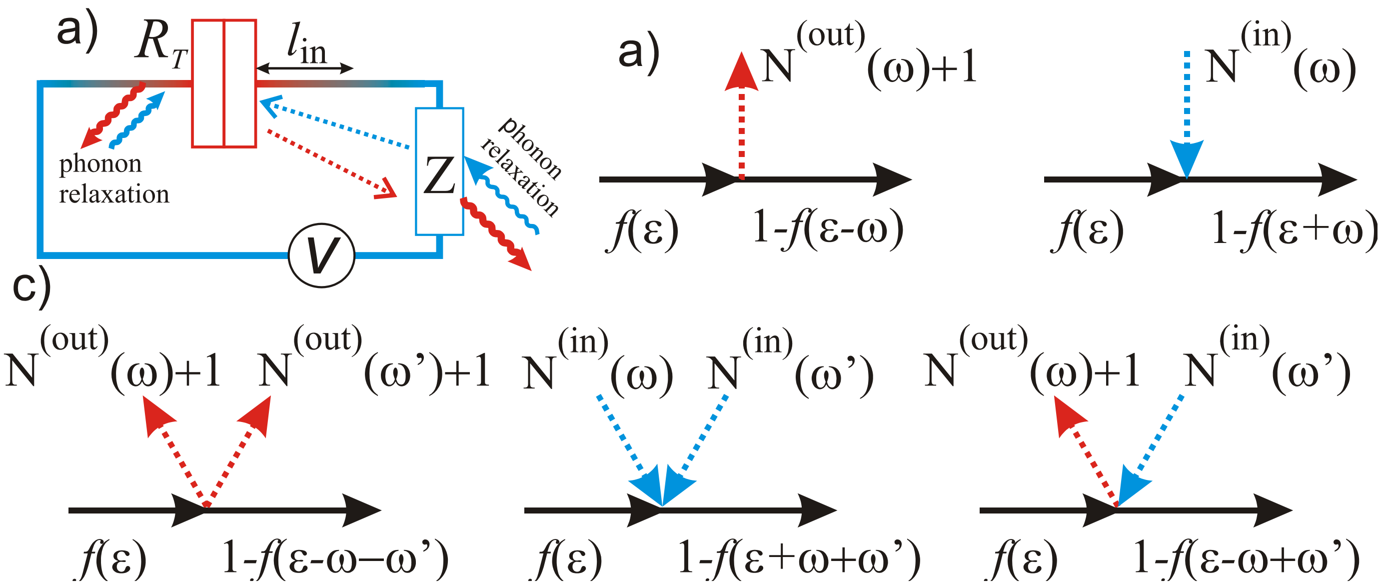
Environment and bath



Environment and bath: two-stage relaxation



Kinetic equations



$$\left(\frac{d}{dt}N_\omega^{(\alpha)}\right)_{\mathrm{e-env}} \sim -\frac{\rho^{(\alpha)}(\omega)}{\nu_\omega^{(\alpha)}R_{\mathrm{T}}}\times$$
$$\left[N_\omega^{(\alpha)}(1+n_\omega^{(\alpha)})-(1+N_\omega^{(\alpha)})n_\omega^{(\alpha)}\right]$$

$$\left(\frac{d}{dt}N_\omega^{(\alpha)}\right)_{\mathrm{e-env}} \sim -\frac{\rho^{(\alpha)}(\omega)}{\nu_\omega^{(\alpha)} R_{\mathrm{T}}} \times \left[N_\omega^{(\alpha)}(1+n_\omega^{(\alpha)}) - (1+N_\omega^{(\alpha)})n_\omega^{(\alpha)}\right]$$

$$n_\omega^{(12)} = \frac{1}{2\omega}\int_\epsilon f^{(i)}_{\epsilon+\omega/2}\sigma^{\mathrm{x}}_{ij}(1-f^{(j)}_{\epsilon-\omega/2})$$

$$1/\tau^{(\alpha)}_{\mathrm{env-e}} = \rho^{(\alpha)}(\omega)/\nu_\omega^{(\alpha)} R_{\mathrm{T}}$$

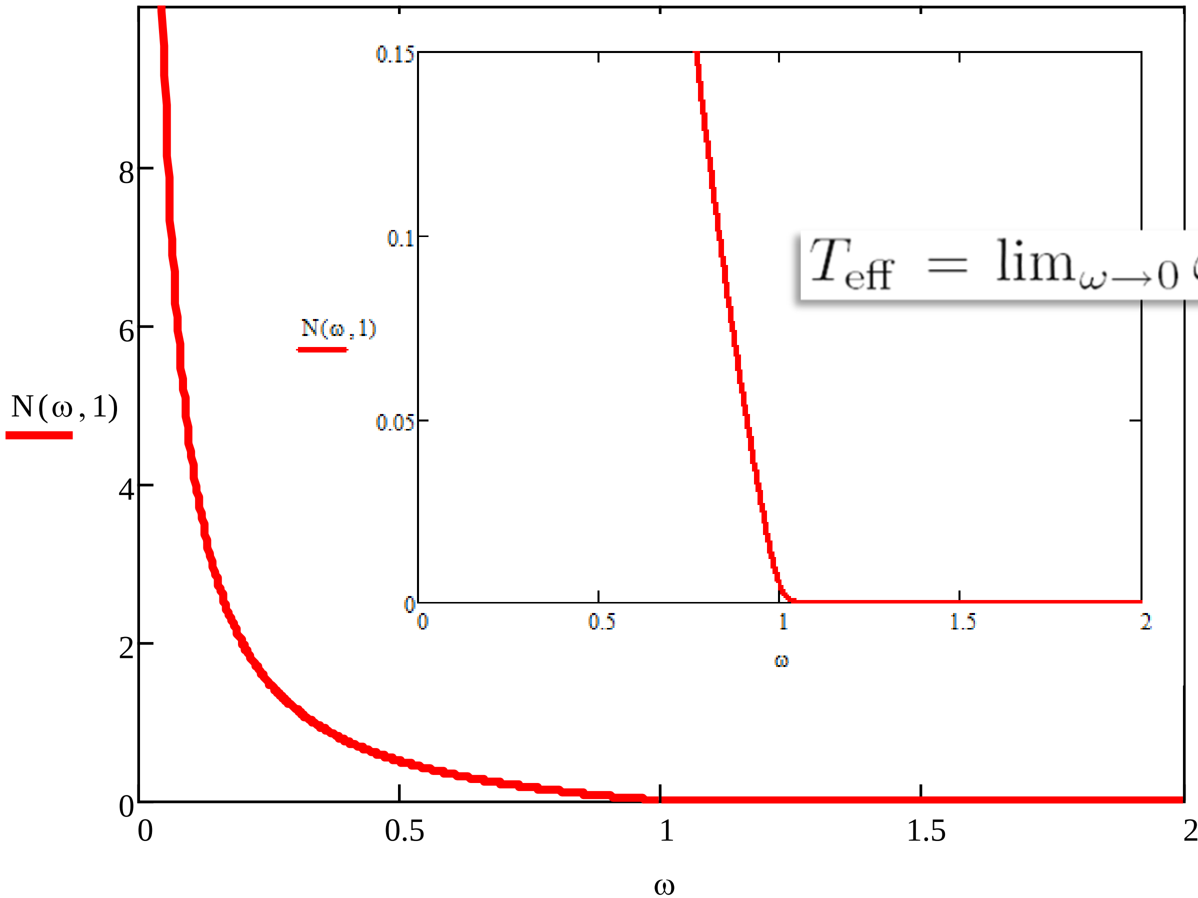
$$\rho_{\alpha=0}(\omega) = \mathrm{Re}[Z_{\mathrm{t}}(\omega)]/R_{\mathrm{Q}}$$

$$\rho_{\alpha\in\{1,2\}}(\omega) = 2\,\mathrm{Im}\int_{\mathbf{q}}(D_\alpha q^2 - i\omega)^{-2}\tilde{U}_\alpha$$

$$\rho_3(\omega) = -2\,\mathrm{Im}\int_{\mathbf{q}}[(D_1 q^2 - i\omega)(D_2 q^2 - i\omega)]^{-1}\tilde{U}_3$$

$$n_B(\omega) := \frac{1}{\exp\left(\frac{\omega}{T}\right) - 1} \qquad T := 0.01$$

$$N(\omega, V) := \frac{1}{2\omega}\cdot\left[(\omega - V)\cdot n_B(\omega - V) + (\omega + V)\cdot n_B(\omega + V)\right]$$



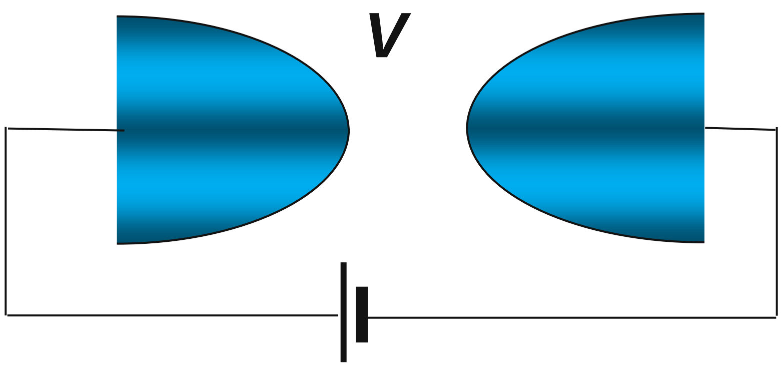
$$T_{\text{eff}} = \lim_{\omega\to 0} \omega N_\omega$$

Examples: *I-V* characteristics of a single junction

Introduce parameters: $g = \rho(0)$; Λ, the characteristic frequency of the $\rho(\omega)$ decay

[for Ohmic model, $\rho = g^{-1}/\{1 + (\omega/\Lambda)^2\}$ and Λ/g is of the order of the charging energy of the tunnel junction]

$$T_{\text{eff}} = \lim_{\omega\to 0} \omega N_\omega$$



T is the temperature in the leads

$$T_{\text{eff}} = 0.5V \coth(V/2T)$$

$$I \sim \frac{V}{R_{\text{T}}} \ln \frac{\Lambda}{V}$$

$$T \ll V \ll \Lambda$$

where $T_{\text{eff}} \simeq V$

Excess voltage

$$I(V/\Lambda \gg 1) \simeq \frac{V - \Delta_\infty}{R_{\mathrm{T}}},$$

$$\Delta_\infty = iJ'(t=0) = 2\int_0^\infty \rho d\omega \{1 + N_\omega^{(\mathrm{out})} - N_\omega^{(\mathrm{in})}\}.$$

In this regime $V > \Lambda$, $N_\omega^{(\mathrm{out})} \sim \Lambda/\omega \gg N_\omega^{(\mathrm{in})}$ and as a result the parameter Δ_∞ acquires the voltage dependence:

$$\Delta_\infty \sim \Delta_\infty^{(0)} \ln(\Lambda/\min\{T_e, T_{\mathrm{env}}\}),$$

$$\Delta_\infty^{(0)} = \Delta_\infty[N^{(\mathrm{out})} = N^{(\mathrm{in})}] = 2\int_0^\infty \rho d\omega \sim \Lambda/g \ll \Delta_\infty.$$

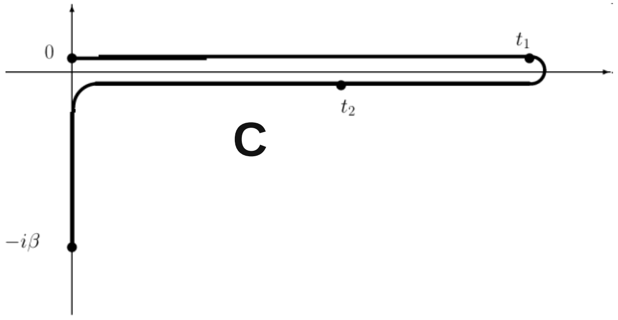
Nonequilibrium physics in superconductors

- in equilibrium physics: $\langle \hat{O}(t) \rangle = \mathrm{Tr}\left\{ \rho \hat{O}_H(t) \right\}$ with Heisenberg operator $\hat{O}_H(t) = \hat{U}(t_0, t)\hat{O}\hat{U}(t, t_0)$

- in non-equilibrium the time evolution needs to be evaluated using the "Keldysh" contour C

  → the generating function is

$$\mathcal{Z} = \int \mathcal{D}m\mathcal{D}\phi \exp\left(-i \int_{\mathcal{C}} dt \left\{ \mathcal{L}[m] + \mathcal{L}_{\text{env}}[m, \phi] + \mathcal{L}\text{int}[\phi] \right\} \right)$$



the non-equilibrium one-particle Green's function is obtained from that for the field operators

N. Chtchelkatchev and V. Vinokur, arXiv:0812.2372 (2009).

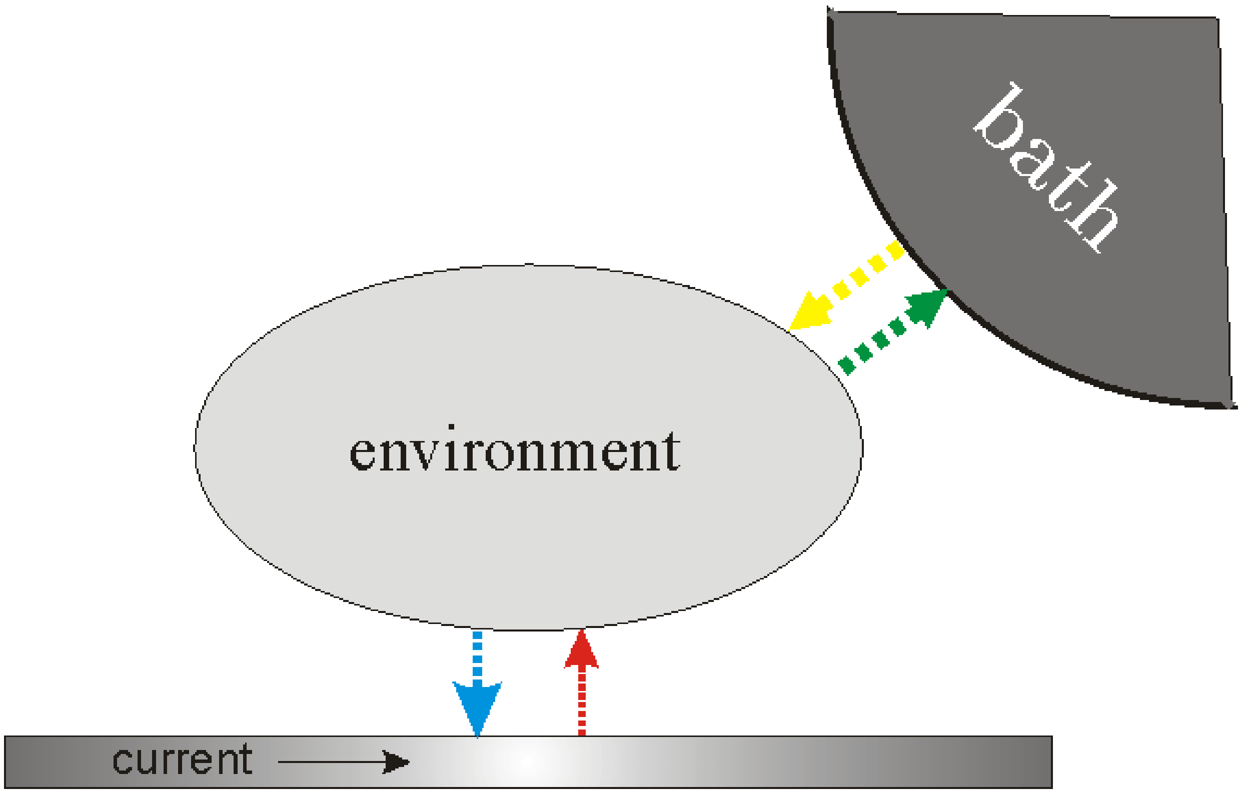
Conclusions

- We have constructed a general theory of far from the equilibrium transport in large arrays of tunneling junctions

- We have derived analytical expression for bias-dependent effective temperature of the environment and found the I-V dependence for a single junction

- We have analyzed several consequences of general theory:
  - Simple derivation of “many body localization” result
  - Microscopic mechanism for insulator-superinsulator transition
  - Resolving the VRH pre-factor puzzle.


probe-1
probe-2
nanosystem
probe-3
probe-4


bath
environment
current

Transport in granular materials: cotunneling

Relaxation mechanism:
creation of the string of the
electron-hole pairs

I. S. Beloborodov, A. V. Lopatin, and V. M. Vinokur
Coulomb effects and hopping transport in granular metals
Phys. Rev. B 72 (12), 125121 - 125141 (2005)

The same mechanism was proposed in
“Many body localization”
I. Gorny, A. Mirlin, D. Polyakov, Phys. Rev. Lett. **95**, 206603 (2005)
D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 76, 052203 (2007).
D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys.(N.Y.) 321, 1126 (2006).